Brazil, Russia, India, China (BRIC) and the West 1950 – 2009: Random Matrix Theory and Business Cycle Synchronisation

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Abstract

The degree of convergence of the business cycles of the BRIC economies (Brazil, Russia, India and China) with the West is an important indicator of the extent to which these economies are synchronised with those of the developed world.

Most of the analytical techniques used in the business cycle convergence literature rely upon the estimation of an empirical correlation matrix of time series data of macroeconomic aggregates. However, the structure of this matrix may be dominated by noise rather than by true information.

Random matrix theory was developed in physics to overcome this problem, and to enable true information in a matrix to be distinguished from noise

We analyse real GDP growth in the BRIC economies and those of the United States, United Kingdom, Germany and Japan over the 1951-2009 period. For much of this period, the movements in GDP of the individual BRIC economies were uncorrelated either with those of the West. However, since the early/mid 1990s, there has been a very marked increase in the degree of synchronisation of the BRIC economies with those of the developed world.

1. Introduction

The four largest emerging economies – Brazil, Russia, India and China (BRIC) – are rapidly becoming increasingly important to the world economy. We examine in this paper the temporal evolution of the convergence of business cycles, both within the BRIC group and between the BRIC countries and the major Western ones - the United States, Japan, Germany and the United Kingdom. This is a key indicator of the degree of synchronisation of the BRIC economies with those of the developed world

We use the technique of random matrix theory ([1]) to analyse the correlations between the GDP growth rates of the economies over time. The use of this technique in this context has been established in previous papers which examined the temporal evolution of the convergence of business cycles in major Western economies (for example, [2, 3, 4]).

Section 2 discusses the relevance of this theory, and section 3 sets out the empirical results.

2. Random matrix theory

The standard way in economics of analysing the degree of synchronisation of the business cycles is to calculate the correlation matrix of the matrix of observations for each economy. This correlation matrix is formed from a time series related to a standard measure of real economic activity – usually the growth in real output (GDP). The literature on business cycle synchronisation has grown substantially over time (recent examples include, [5, 6, 7, 8]).

If $\underline{\underline{M}}$ is an N x T rectangular matrix (T observations of the GDP growth of the N economies) and $\underline{\underline{M}}^T$ is its transpose, the correlation matrix $\underline{\underline{C}}$ as defined below is an N x N square matrix

$$\underline{\underline{C}} = \frac{1}{T} \underline{\underline{M}}\,\underline{\underline{M}}^T$$

However, due to the finite size of N (which corresponds to the number of economies) and T (which is the number of observations of GDP) then a reliable determination of the correlation matrix may prove to be problematic. The structure of the correlation matrix may be dominated by noise rather than by true information.

In order to assess the degree to which an empirical correlation matrix is noise dominated we can compare the eigenspectra properties of the empirical matrix with the theoretical eigenspectra properties of a random matrix. Undertaking this analysis will identify those eigenstates of the empirical matrix who contain genuine information content. The remaining eigenstates will be noise dominated and hence unstable over time. This technique has been applied by many researchers to financial market data (for example, [9, 10, 11, 12, 13, 14]).

For a scaled random matrix **X** of dimension N x T, (i.e where all the elements of the matrix are drawn at random and then the matrix is scaled so that each column has mean zero and variance one), then the distribution of the eigenvalues of the correlation matrix of **X** is known in the limit T, N $\rightarrow \infty$ with $Q = T/N \geq 1$ fixed, as in [15]. The density of the eigenvalues of the correlation matrix, λ, is given by:

$$\rho(\lambda) = \frac{Q}{2\pi} \frac{\sqrt{(\lambda_{max} - \lambda)(\lambda - \lambda_{min})}}{\lambda} \qquad \text{for } \lambda \in [\lambda_{min}, \lambda_{max}] \qquad (1)$$

and zero otherwise, where $\lambda_{max} = \sigma^2 (1 + 1 / \sqrt{Q})^2$ and $\lambda_{min} = \sigma^2 (1 - 1 / \sqrt{Q})^2$ (in this case $\sigma^2 = 1$ by construction).

The eigenvalue distribution of the correlation matrices of matrices of actual data can be compared to this distribution and thus, in theory, if the distribution of eigenvalues of an empirically formed matrix differs from the above distribution, then that matrix will not have random elements. In other words, there will be structure present in the correlation matrix.

In terms of any eigenvalues which lie outside the noisy sub-space band the most important from a macroeconomic perspective is the largest eigenvalue. The application of these techniques to equities traded in financial markets has demonstrated that this eigenmode corresponds to the 'market' eigenmode (e.g. [12]). In this context the largest eigenvalue will inform us as to the degree to which the movements of the emerging economies are correlated.

The trace of the correlation matrix is conserved, and is equal to the number of independent variables for which time series are analysed. So, for example, considering the four BRIC economies together, the trace is equal to 4 (since there are 4 time series). The closer the 'market' eigenmode (i.e. eigenmode 1) is to this value the more information is contained within this mode i.e. the more correlated are the movements of GDP.

The market eigenmode corresponds to the largest eigenvalue. The degree of information contained within this eigenmode, expressed as a percentage, is therefore $100\lambda_{max}/ N$. To follow the evolution of the degree of business cycle convergence over time we may analyse how this quantity evolves temporally.

The analysis is undertaken with a fixed window of data. Within this window the spectral properties of the correlation matrix formed from this data set are calculated. In particular the maximum eigenvalue is calculated. This window is then advanced by one period and the maximum eigenvalue noted for each period.

3. The data and the results

For all eight countries, annual levels of real GDP over the period 1950-1994 are available from Maddison [16]. We obtained the more recent data from the International Monetary Fund (IMF) global database[1].

We look at the correlation matrix of real GDP growth rates for both the rapidly advancing economies group and the developed major economies group. In addition, we analyse the correlation matrix for various permutations of these economies.

For the full data set of 59 annual observations of GDP growth in the BRIC economies, the eigenvalues of the correlation matrix are in the range 0.50 to 1.46. The theoretical range of the eigenvalues of a random matrix of the same dimension is 0.55 to 1.59. These results indicate the presence of a large amount of noise in the correlation matrix.

The economic implication of this result is that it is not meaningful to speak of a business cycle between the BRIC economies existing over the 1950-2009 period as a whole. The individual countries experienced booms and busts, but their cycles were not correlated. This is not really surprising. For much of the period, the Russian economy operated under the centrally planned regime of the former Soviet Union. China and India, too, were both oriented towards central planning rather than the market-oriented structures of the West. It is only recently that the BRIC economies in general have begun to engage with the major economies of the West.

Even amongst the latter, for much of their history, the degree of synchronisation between their individual business cycles has generally been low ([17] and [4]). It is only really during the period since the first oil shocks of 1973/74 that we can meaningfully speak of an international business cycle in the West.

[1] Data for 2009 are the latest estimates

Analysing the temporal evolution of the business cycles of the BRIC economies, however, shows that the degree of synchronisation between these economies has moved rather dramatically in recent years.

The choice of an appropriate window to span the periodicity of what constitutes the business cycle is not completely straightforward. Business activity is influenced by a very large number of events, and these events may be very diverse in character and scope. Individual cycles therefore vary both in terms of amplitude and period. This lack of regularity may be analysed formally using random matrix techniques ([18]). The evidence for the existence of a business cycle at all relies more upon factors such as the fact that output changes in different sectors of an economy tend to move together (as shown by Lucas [19]) than upon regularities in either amplitude or period of the economy as a whole.

A major study of the US economy ([20]) many years ago concluded that the period ranged from some two to twelve years, a range which still commands broad assent amongst economists. For the UK, the estimated range is 7 to 12 years and for Germany, for example, 6 to 12 years. Again, the concentration at these frequencies is only weakly determined.

The results presented in this paper use a window of ten years, although they are robust to using windows of both eight and twelve years. Full details of the calculations are available on request from the authors.

Figure 1 sets out the temporal evolution of business cycle synchronisation between the BRIC economies over the 1950-2008 period, using a window of ten years. Each window contains 10 annual observations, and so there are 50 windows in total. The period 1951 - 1960 corresponds to the first data point in Figure 1, 1952 - 1961 to the second, and so on through to 2000 - 2009. The chart shows the evolution over time of $100\lambda_{max}/ N$, where λ_{max} is the largest eigenvalue of the correlation matrix of annual real GDP growth rates.

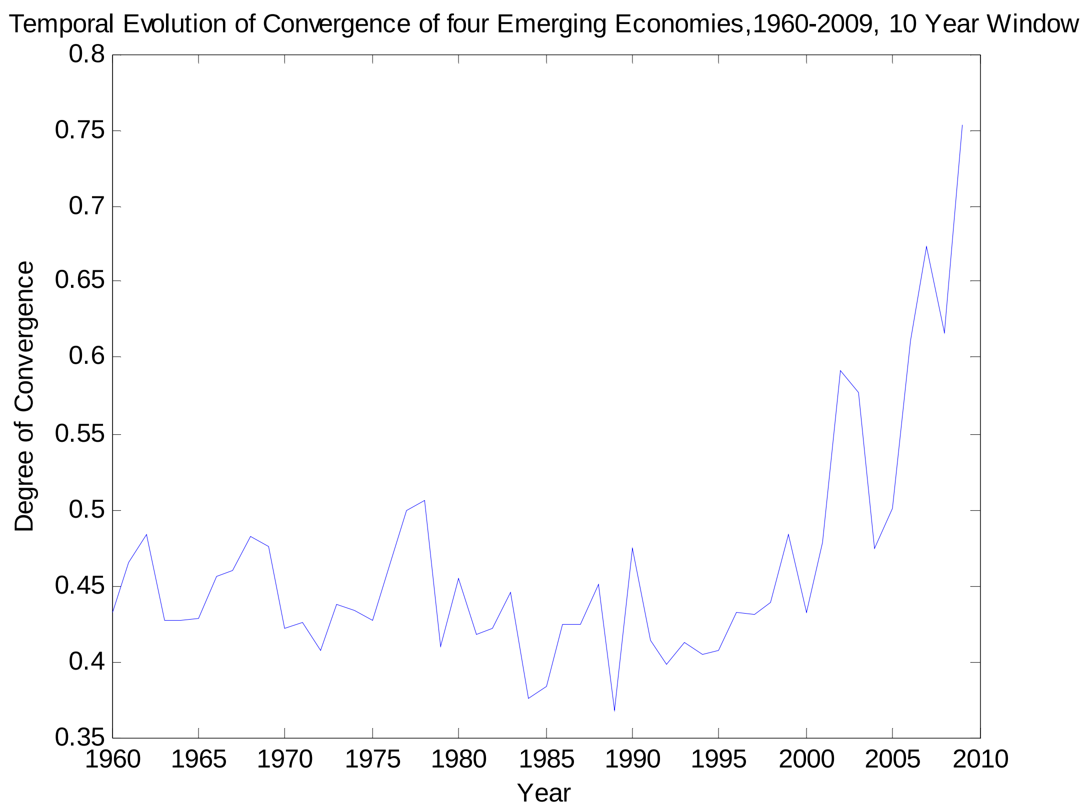

Figure 1

The temporal evolution of the degree of information content in the maximum eigenvalue of the empirical correlation matrix formed from the time series of annual GDP growth for the economies of Brazil, Russia, India and China. Each window of data spans 10 annual observations. The period 1951-1960 corresponds to the first data point in Figure 1, 1952-1961 to the second, and so on through to 2000-2009.

The level of convergence between the four emerging economies is low until around the point marked '2001' in the chart. This corresponds to the period 1992-2001. However, the economies have converged substantially in recent years, the principal eigenvalue over the 2000-2009 period accounting for some 75 per cent of the total value of the eigenvalues. Over this period, there is some genuine correlation between the GDP growth rates in the four economies, the largest eigenvalue being 3.01 compared to the maximum value for a random matrix calculated from (1) of 2.66

This indicates a dramatic increase over the past two decades in the degree of synchronisation of the business cycles of the BRIC economies.

Figure 2 sets out the same calculations for the four major Western economies considered here - the US, UK, Germany and Japan.

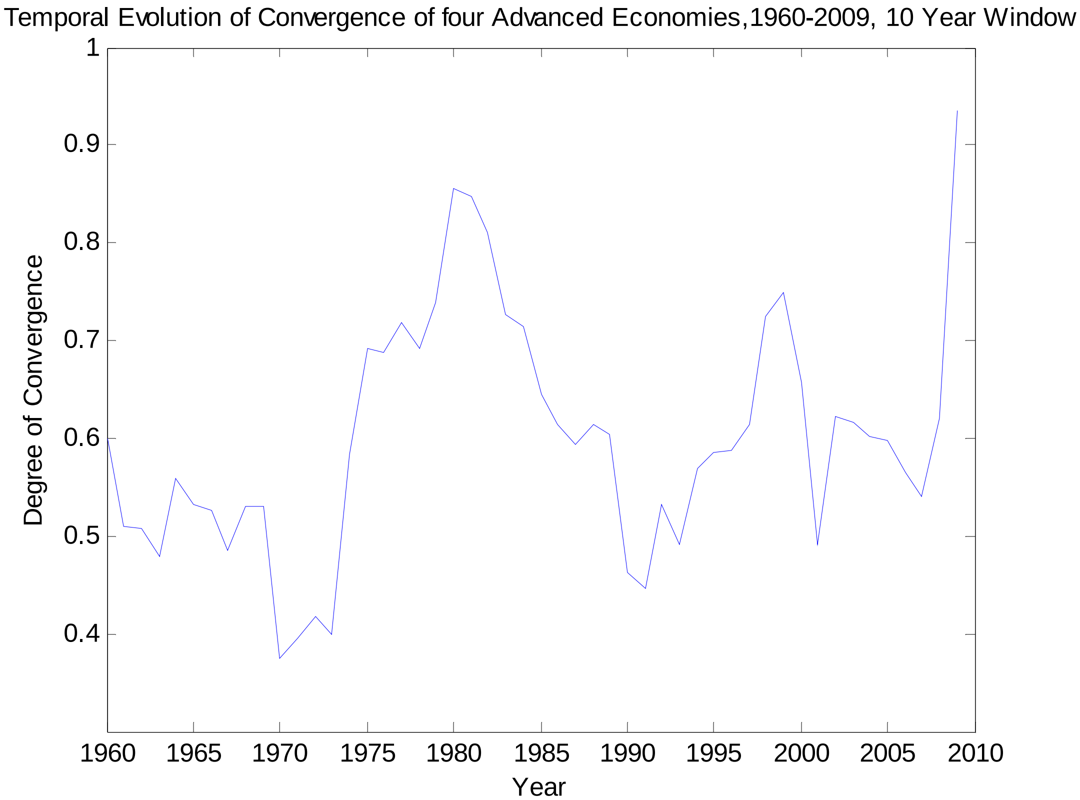

Figure 2

The temporal evolution of the degree of information content in the maximum eigenvalue of the empirical correlation matrix formed from the time series of annual GDP growth for the economies of the United States, United Kingdom, Germany and Japan. Each window of data spans 10 annual observations. The period 1951-1960 corresponds to the first data point in Figure 1, 1952-1961 to the second, and so on through to 2000-2009.

Although the synchronisation between the Western economies was higher in general than in the BRIC until the mid-1970s, this was not markedly the case. Remember that the point '1975' in Figure 2 corresponds to the period 1966-1975. It is only subsequent to then that marked convergence of the individual business cycles takes place, as we noted above. For example, over the 1951-60 period, the maximum eigenvalue is 2.40 compared to the theoretical maximum of a random matrix of 2.66, whereas for 2000-2009, the maximum eigenvalue is 3.75.

We now proceed to add the BRIC economies one by one to those of the four major Western economies and to analyse the outcome. Figure 3 plots, for illustration, $100\lambda_{max}/N$ for the four major economies and for the four plus Russia.

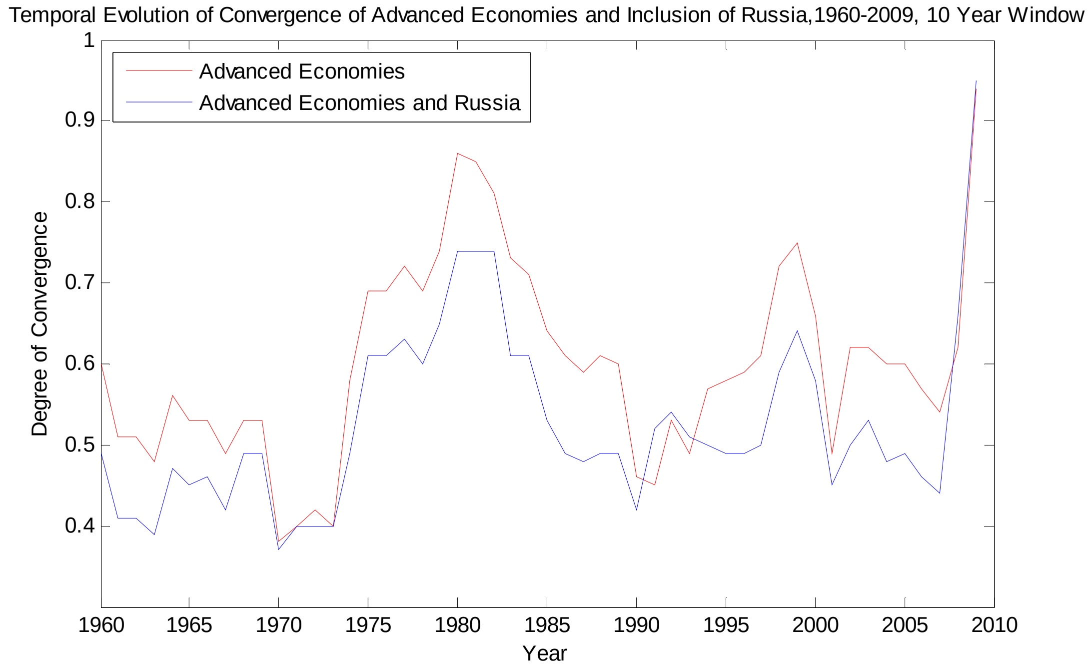

Figure 3

The temporal evolution of the degree of information content in the maximum eigenvalue of the empirical correlation matrix formed from the time series of annual GDP growth for the economies of the United States, United Kingdom, Germany, Japan and Russia. Each window of data spans 10 annual observations. The period 1951-1960 corresponds to the first data point in Figure 1, 1952-1961 to the second, and so on through to 2000-2009.

Although the overall shape of the plots with and without Russia is very similar, including Russia in the group weakens the degree of convergence between the economies. This is a significant weakening, as a formal Kolmogorov-Smirnov test shows. The null hypothesis that the distribution of $100\lambda_{max}/ N$ is the same both including and excluding Russia is rejected at a p-value of 0.000.

The null hypothesis is also rejected for the three other BRIC economies when we add them to the Western group individually. For Brazil, it is rejected at p = 0.004, for China 0.009 and India 0.000.

For completeness, Figure 4 plots the results of adding each of the BRIC economies individually to the Western group

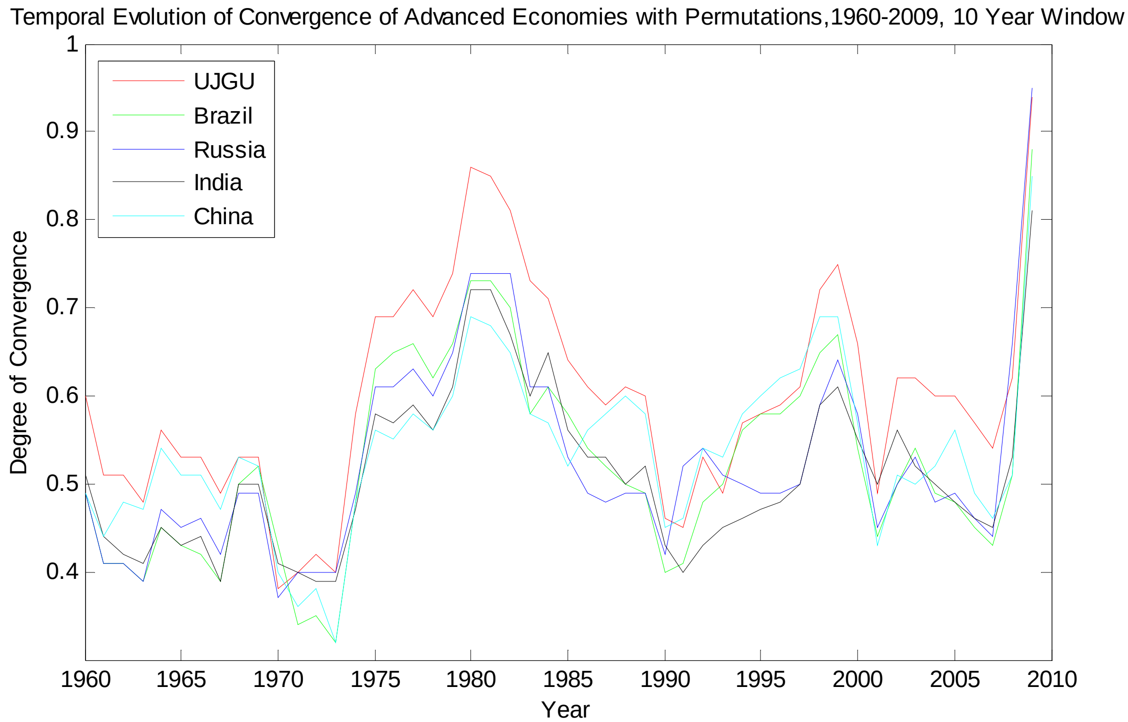

Figure 4

The temporal evolution of the degree of information content in the maximum eigenvalue of the empirical correlation matrix formed from the time series of annual GDP growth for the economies of the United States, United Kingdom, Germany, Japan (UJGU) and the BRIC economies. Each window of data spans 10 annual observations. The period 1951-1960 corresponds to the first data point in Figure 1, 1952-1961 to the second, and so on through to 2000-2009.

Qualitatively, the past decade or so has seen an increase in the business cycle synchronisation of the BRIC economies with those of the West, beginning with the period 1998-2007. However, the economies are still not completely aligned. For example, over the 2000-2009 period for the four Western economies, $100\lambda_{max}/ N$ takes the value 0.94. Adding all the BRIC countries gives a value of 0.85, indicating a strong degree of convergence within this wider group of eight economies, but still not as strong as purely within the Western group.

4. Conclusion

In this paper, we analyse the convergence or otherwise of the business cycle in the main Western economies of the United States, United Kingdom, Germany and Japan and the major developing economies, the BRIC group of Brazil, Russia, India and China. We examine annual rates of growth of real GDP over the 1950-2009 period. The correlations between the growth rates are analysed using random matrix theory, which enables us to identify the extent to which the correlations contain true information rather than noise.

Not surprisingly, within the BRIC group itself there was essentially no correlations of their business cycles during the 1950s, 1960s, 1970s and 1980s. Since the early 1990s, however, there has been a very marked rise in the degree of synchronisation.

The integration with the West has also risen sharply, though with a lag of a few years, the degree of synchronisation is showing sharp increases from the mid-1990s onwards. The synchronisation between the BRIC economies and the West is still not as strong over the past decade or so as it is between the major Western economies themselves, but it is nevertheless high. For the first time, in 2009 it is meaningful to speak of a global economic cycle.

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