Scaling invariant distributions of firms' exit in OECD countries*

Corrado Di Guilmi[a]
Mauro Gallegati[a]
Paul Ormerod[b]

[a]*Department of Economics, University of Ancona, Piaz.le Martelli 8, I-62100 Ancona, Italy*
[b]*Volterra Consulting, 121 Mortlake High Street, London SW14 8SN, UK*

Abstract
Self-similar models are largely used to describe the extinction's rate of biological species. In this paper we analyse the extinction's rate of firms in 8 OECD countries. Firms are classified by industrial sectors and sizes: we find that while a power law distribution with exponent close to 2 fits very well the extinction rate by sector, a Weibull distribution is more appropriate if one analyses firms' size.

1. Introduction

Power laws are definitely not new in economics.[1] Only recently scaling plot techniques have been applied to research on firms' extinction rate, the main examples being [9] (henceforth, CO), who present evidence of power law scaling for the demise of US firm firms in the long run. In particular, CO show that the exit rate follows approximately a power law distribution with exponent close to 2. This value is very much in line with the literature on the Raup-Sepkoski's kill curve according to which biological extinction events "can be reasonably well fitted to a power law with exponent between 1 and 3" ([10] p.165). This suggests that the mainstream economic model (i.e. the General Economic Equilibrium approach) may be incomplete or inadequate, since it assumes that the distribution of observations depends on the scale.[2]
Here we aim at extending and controlling for robustness the CO findings. Differently from CO, who limit their empirical analysis to the US data, we use an OECD data set of eight industrialized countries.[3] Using a larger country data set allows us to get rid of the country-specific evidence which may affect the findings. Second, rather than taking an historical perspective by using data spanning well over one century we use a very short term perspective, because data are much reliable.
The paper is organised as follows. In the second section we classify firms by industries obtaining a total of 5051 observations and treat them as a sum of daily uncorrelated events: their distribution follows a power law with exponent close to 2. In section 3 we classify firms by size: In this case

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* G.Giulioni and A.Palestrini provided very useful comments.
[1] Topics include income distribution [1], returns on financial assets [2], firm sizes [3], city sizes [4], revenues in the motion picture industry [5] and business cycles [6-8].
[2] [11] emphasizes that scale-free behaviour are connected to agents' interaction, while [12] points out the failure of neoclassical theory of firms in explaining at what level agents behaviour can be analysed autonomously from the market.
[3] [13]: the countries are Denmark, Finland, France, Italy, Netherlands, Portugal, United Kingdom and United States.

their distribution in best fitted by a Weibull curve, which is not surprising at all, since small (large) firms are more (less) likely to exit the market.

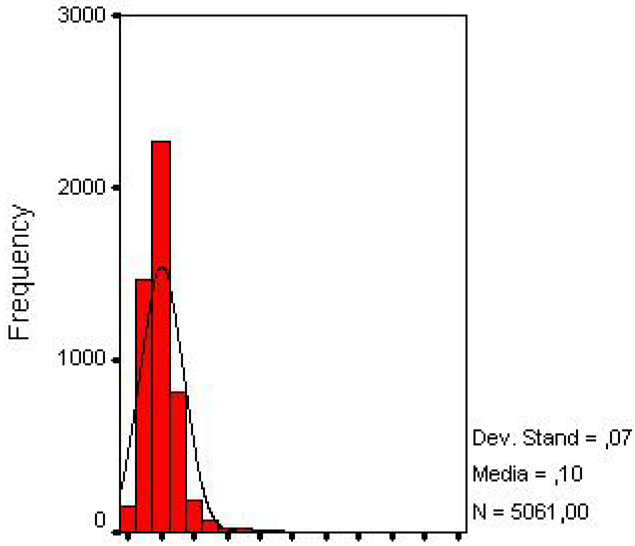
2. Distribution by industry

Literature on extinctions of biological species [14, 15] shows that exit's rate obeys a power law with exponent close to 2. CO find that the frequency of extinction[4] (in proportion on the total of existing firms) of USA firms (divided by years, state and industrial sector) can be approximated by the same distribution. In this section we try apply the same methodology to a different data set, regarding demises of firms in eight OECD countries in the period 1977-1999, for a total of 5051 observations.

Each observation of the population is the sum of independent and identically distributed 250 random variables (being 250 the approximate number of working days per year) and, since they converge only very slowly to a Gaussian distribution, one can assume that the underlying distribution is a power law with finite variance [14]. As a first step we verify the independence of extinctions. Although we do not get univocal indication, data do not show a relevant correlation at any level of disaggregation, being:

- 0.19 that relative to the average percentage of demises for *country* with the precedent year;
- 0.6 across the *industrial sector* in each country, and
- 0.15 across *sectors and countries*.

Fig. 1: Extinctions (on total of firms)



The Kolmogorov-Smirnov test rejects the hypothesis that the distribution does converge to a normal distribution with probability p=0.00. To test the hypothesis of a power law distribution of the daily observations we generate a simulate distribution from the actual daily data, drawing a 250 values sample (for each annual data) from the following probability distribution:

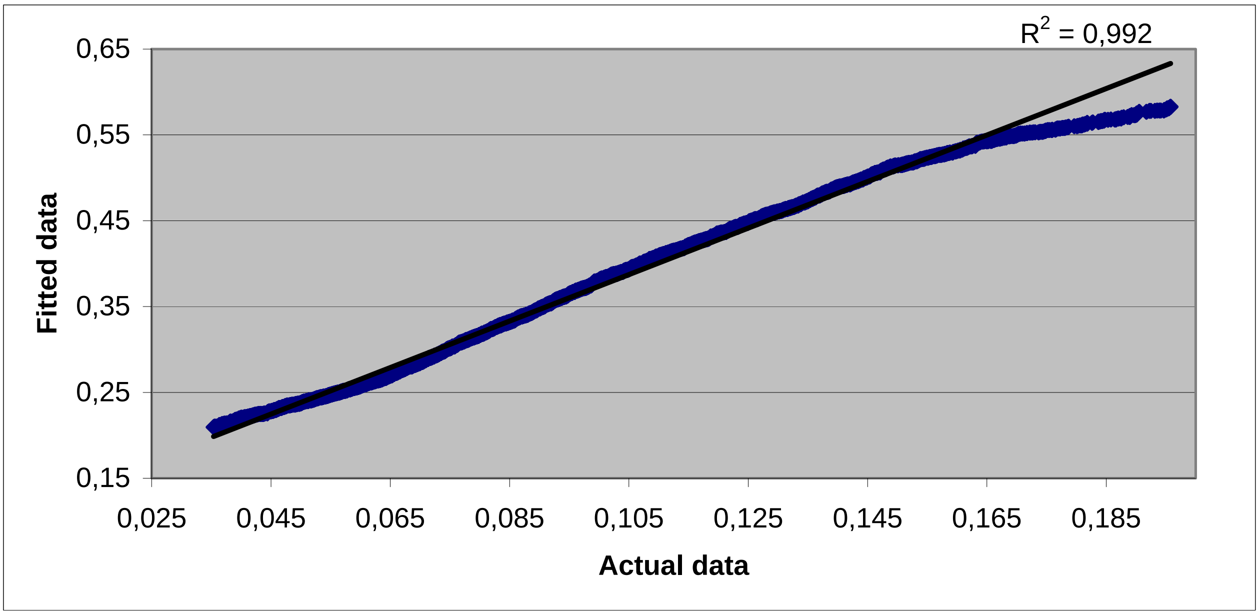
[4] According to the literature [16, 17] firms "die" because of merging, voluntary liquidation, or bankruptcy. According to the OECD data set disappeared firms are those that leave national business or social security registers.

$$P(X=i)=\frac{(i+1)^{-r}}{\sum_{j=0}^{m}(j+1)^{-r}}$$

with i,j=0,…, m, where *m* is the maximum of daily demises.

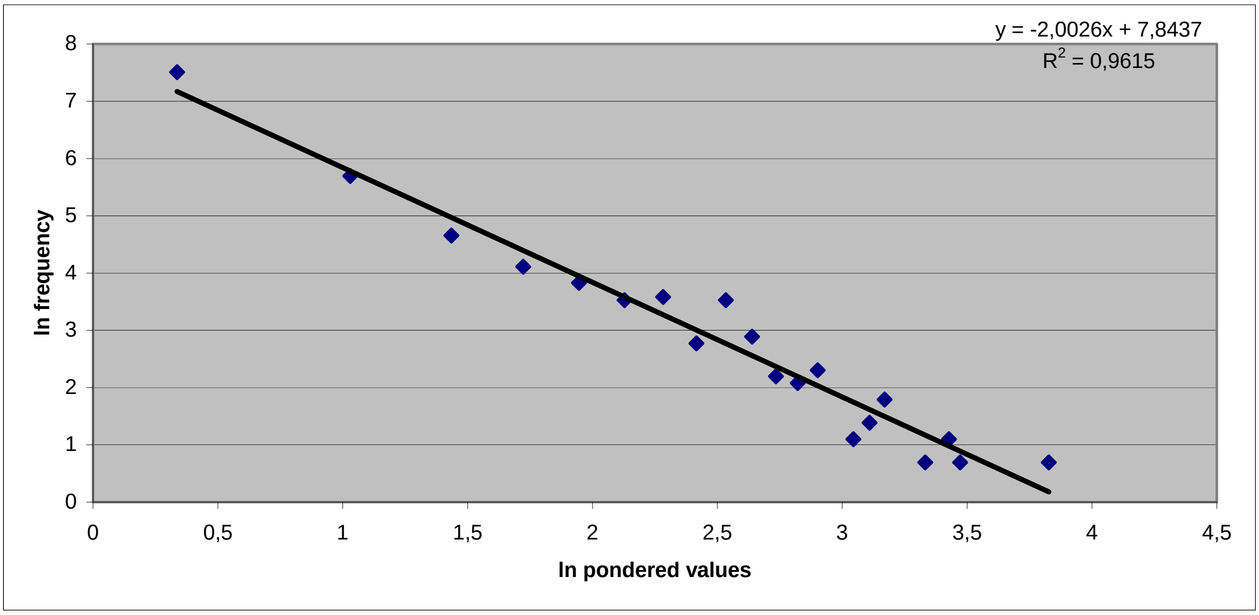
After aggregating the drawn values to obtain a series of 5051 annual data we plot them against the actual data, varying r and m on an enough wide range. Parameter m allows to set the variance of the distribution to a finite value (otherwise we should admit the theoretical possibility of limitless demises per year). The highest correlation is obtained with r=2 and m= 400 , with a regression coefficient of 0.992 (figure 2).

Figure 2



The two distributions are very different, as confirmed by Kolmogorov-Smirnov test which yields a probability value of 0.00. The tail has a good fit, but it tends to overestimate the upper part of the distribution and to underestimate the lower. Thus, in order to test the robustness of the so obtained results, we report here following another figure (figure 3) in which we plot, in a log-log plane, the absolute values of annual demises, pondered over their relative frequency, on their absolute frequency. The interpolation line, also in this way, reports a decay coefficient very close to two.

Figure3



Analogously to what CO find for the USA case, we can show that the OECD distribution of demises of firms is analogue to the one observed for the extinction of biological species with a coefficient of 2.

3. Distribution by size

Another interesting figure regards the correlation between distribution of extinction rates and sizes of firms involved. Intuitively greater firms have less chance to leave the market than the smaller ones [18]. The data set we examined reports the demises sorted by number of employers of the firm[5] for a total of 548 observations for 9 countries[6] (each observation is individuated by year, country and size.) Variation rates in aggregate are not highly correlated with previous year's one, while disaggregated data show a strong dependence among the same size class (we have no indication about the behaviour at sub-year level.) The Kolmogorov-Smirnov test rejects the hypothesis that the observation are normally distributed at 100% probability level (even taking only the 5 percent of the values that are just around the mean the result does not change.)
This section analyses the sub-distribution of demises (in their absolute values) over the size thresholds and approximates them by a Weibull distribution. The Weibull CDF takes the form:

$$F(x) \equiv 1 - \exp\left(-\alpha x^{\beta}\right) \qquad (1)$$

where α is the scale parameter and β is the characteristic shape parameter that quantifies the rapidity of decay of the distribution [19].
Ordering the n observations from largest to smallest and indicating with x_i the i^{th} observation ($x_1 > x_2 > \ldots > x_n$), one has:

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[5] The firms are sorted in six classes (0, under 20, between 20 and 49, between 50 and 99, between 100 and 499, more than 500).
[6] In this section we extend the analyse West Germany also, which was not included in the previous section because of the absence of data relative to the first dimensional class that may cause distortions in the aggregate (and thus in the percentage) of demises in that country.

$$\frac{i}{n} \equiv 1 - F(x_i) \qquad (2).$$

Substituting (1) in (2) and taking the natural logarithm yields:

$$x^\beta \equiv -\frac{1}{\alpha}\ln(i) + \frac{\ln(n)}{\alpha}$$

Setting:

$$\frac{\ln(n)}{\alpha} \equiv \varphi \quad \text{and} \quad \frac{1}{\alpha} \equiv \lambda ,$$

we get:

$$x^\beta \equiv -\lambda \ln(i) + \varphi \qquad (2),$$

that is the interpolation line on the semi-log plane, where we plot *x* taken at exponent β on the natural logarithm of its rank.
We get a good fit for every class (Rsq is almost more than 0.96 in every case) even if the tails tend to overestimate highest rates. Table 1 reports values of distributions parameters and the slope of regression line.

Table 1: parameters of weibull distributions.

	α	β	λ
>0	7,0448	1,1469	6,2795
>20	6,0498	1,2989	5,1357
>50	4,36	1,164	3,6518
>100	3,0555	0,9978	2,5500
>500	2,022	0,787	1,6909

At value of β equal to 1 the distribution is a common exponential, that become flatter as β decrease; while, for β>1, observations are concentrated around the mode and their distribution may approximate the normal. Both parameters decrease as we eliminate from the range of observation the lower size classes. The trend of the shape parameter β evidences that the tail becomes flatter, as also indicated by the slope of the regression line.
This may be an effect of the reduction of the number of observations as we move toward the upper size classes, but it also reveals that, considering a wide range of sizes, decay of distribution is faster, especially around the mode, while, discarding lower size classes, the scale become less sharp and observations are more dispersed (there is no mode). The decrease of the scale parameter confirms the flattening of distribution curve.

4. Conclusions

In this paper we show that extinctions of firms (considering frequency of extinction events as function of its size) in the 8 of the OECD [13] data set are very well fitted by a power law. Intriguingly, the estimated exponent of distribution is the same reported for biological species.

Analysing frequency of exit as dependent of firms' size, that are better fitted by Weibull distribution, we find that events in lower dimensional classes tend to exhibit a more regular behaviour, being more concentrated around the mode. Given the nature of our cross section analysis, the amplitude of extinction events seems to be not dependent by country, year or number of existing firms.

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